Equi-homogeneity, Assouad dimension and non-autonomous dynamics

Nicholas Sharples
University of Middlesex

Abstract

The scaling of 'local covers' can vary dramatically between different point of a set. The Assouad dimension is the global upper bound on this scaling. However, even sets that scale identically at every point, such as generalised Cantor sets, can have Assouad dimension strictly greater than their box-counting dimension.

To further the investigation of the Assouad dimension we introduce a new notion of regularity: a set is said to be equi-homogeneous if the scaling of local covers is uniform across all points of the set. Equi-homogeneity is less restrictive than Ahlfors-David regularity as it can be enjoyed by sets with unequal dimensions.

We show that the equi-homogeneous sets include large classes of Moran sets and attractors of both autonomous and non-autonomous iterated function systems, which simplifies the calculation of their Assouad dimension. Further, we show that the Assouad dimension of equi-homogeneous sets can be recovered from the more straightforward box-counting dimensions, provided that the box-counting dimensions are suitably 'well behaved'.