Irregular PDEs, well-posedness and fractal geometry

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Abstract

In this talk I will establish the well-posedness of the transport equation in the case when the coefficients are irregular (e.g. have only Sobolev regularity) and additionally have 'geometrically complicated' sets of singularities. We will see that the well-posedness results hold when the set of singularities has a sufficient small anisotropic fractal dimension, which is encoded in a 'codimension print'. I will relate this esoteric notion of dimension to the more familiar box-counting dimensions to provide straightforward criteria for the well-posedness of the transport equation.
Further, I will consider some qualitative properties of the 'generalised' flow solution of the corresponding ODE, which defines a dynamical system in this irregular setting. This work extends the renormalization theory of DiPerna & Lions and Ambrosio to account for geometrically complicated singularities.