Fourier transforms and Minkowski's question mark function

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Abstract

Minkowski's question mark function is a Holder continuous bijection from the unit interval to itself which maps quadratic irrationals to rationals and can be defined using the continued fraction expansion. It is a singular function and so has 0 derivative almost everywhere (despite being strictly increasing). We will show that it crops up in dynamical systems through the topological conjugacy between the Farey map and the doubling map and as an invariant measure for the Gauss map. A natural question to ask about singular functions is how their Fourier coefficients behave and in fact Salem asked whether the Fourier coefficients for the Minkowski question mark function decay as n tends to infinity. We will show that by viewing the Minkowski question mark function as an invariant measure for the Gauss map this question can be settled. If time permits we'll discuss some consequences of the Fourier transform of a singular measure decaying polynomially. This is joint work with Tuomas Sahlsten (Jerusalem).