THE β -TRANSFORMATION WITH A HOLE

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ABSTRACT. Let $\beta \in (1, 2)$ and consider the β -transformation

 $T_{\beta}(x) = \beta x \pmod{1}.$

Take some interval $(a, b) \subset [0, 1)$ and call this a hole. Then we study the set of points whose orbits do not fall into the hole:

 $\mathcal{J}_{\beta}(a,b) := \{ x \in (0,1) : T^n_{\beta}(x) \notin (a,b) \text{ for all } n \ge 0 \}.$

If the hole (a, b) is large, then 'most' orbits should fall in and so $\mathcal{J}_{\beta}(a, b)$ should be small, and vice versa. In this talk we will use symbolic dynamics and combinatorics on words to describe precisely the relationship between the hole (a, b) and the size of $\mathcal{J}_{\beta}(a, b)$.