## THE $\beta$-TRANSFORMATION WITH A HOLE

LYNDSEY CLARK

Abstract. Let $\beta \in(1,2)$ and consider the $\beta$-transformation

$$
T_{\beta}(x)=\beta x \quad(\bmod 1) .
$$

Take some interval $(a, b) \subset[0,1)$ and call this a hole. Then we study the set of points whose orbits do not fall into the hole:

$$
\mathcal{J}_{\beta}(a, b):=\left\{x \in(0,1): T_{\beta}^{n}(x) \notin(a, b) \text { for all } n \geq 0\right\} .
$$

If the hole $(a, b)$ is large, then 'most' orbits should fall in and so $\mathcal{J}_{\beta}(a, b)$ should be small, and vice versa. In this talk we will use symbolic dynamics and combinatorics on words to describe precisely the relationship between the hole $(a, b)$ and the size of $\mathcal{J}_{\beta}(a, b)$.

