Optimal control of Fourier multipliers by maximal operators

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Abstract

Given an operator T, is it possible to control it by another operator $\mathcal{M}$ having an explicit geometric character? In the first part of this talk, we give an overview of what is known for classical operators in Harmonic Analysis, such as the Hardy–Littlewood maximal function, the Hilbert transform or the Carleson operator, all of them related to the classical problem of convergence of Fourier series. Also, we provide new optimal control in the case of the Carleson operator.

In the second part of the talk, we focus in operators which have an oscillatory nature, and we obtain optimal control for certain classes of oscillatory Fourier multipliers. This is motivated by a long-standing conjecture of E.M. Stein, which states that the disc multiplier is controlled by some variant of the Nykodim (Kakeya) maximal function via an L^2-weighted inequality. Our results also apply to solution operators for dispersive PDE, such as the time-dependent free Schrödinger equation. This is joint work with Jonathan Bennett.

The talk will be aimed for a broad analysis audience.