Projections of planar Mandelbrot measures

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Abstract

Let μ be a planar Mandelbrot measure and $\pi_*\mu$ its orthogonal projection on one of the main axes. We study the thermodynamic and geometric properties of $\pi_*\mu$. We first show that $\pi_*\mu$ is exactly dimensional, with $\dim(\pi_*\mu) = \min(\dim(\mu), \dim(\nu))$, where ν is the Bernoulli product measure obtained as the expectation of $\pi_*\mu$. We also prove that $\pi_*\mu$ is absolutely continuous with respect to ν if and only if $\dim(\mu) > \dim(\nu)$, and find sufficient conditions for the equivalence of these measures. Our results provides a new proof of Dekking-Grimmett-Falconer formula for the Hausdorff and box dimension of the topological support of $\pi_*\mu$, as well as a new variational interpretation. We obtain the free energy function $\tau_{\pi_*\mu}$ of $\pi_*\mu$ on a wide subinterval $[0, q_c)$ of $\mathbb{R}_+$. For $q \in [0, 1]$, it is given by a variational formula which sometimes yields phase transitions of order larger than 1. For $q > 1$, it is given by $\min(\tau_\nu, \tau_\mu)$, which can exhibit first order phase transitions. This is in contrast with the analyticity of τ_μ over $[0, q_c)$. Also, we prove the validity of the multifractal formalism for $\pi_*\mu$ at each $\alpha \in (\tau'_{\pi_*\mu}(q_c-), \tau'_{\pi_*\mu}(0+)]$.

This is joint work with De-Jun Feng.